Non-linear Uniform Colour Space Considering Non-linearity and Non-symmetry in Opponent Colour Response Mechanisms

Koichi IKEDA
Tokyo Rika Daigaku, Department of Electrical Engineering, Faculty of Science and Technology

ABSTRACT

A new non-linear uniform colour space NC-IIIC is developed for colour specification. First, NC-IIIC space is derived by applying non-linearity at the receptor level to linear colour space NC-I composed of linear transformations of tristimulus values \( X, Y \) and \( Z \). Then, NC-IIIC space is constructed by further introducing non-linear and non-symmetric functions in Y-B and R-G opponent colour response mechanisms to NC-IIIC space. For colours aligned on a plane of constant value in Munsell space, coefficients of the above non-linear response functions are optimized by computer numerical analyses to make hue circles shape as close as possible to uniform circles in the new space. The uniformity of the space is significantly improved, and the average deviation between hue circles and uniform circles in the new space is reduced to about 1/10-1/20 of those in conventional \( L^*a^*b^* \) and \( L^*u^*v^* \) uniform colour spaces by applying linear opponent transformations. Hue, lightness and chroma can be represented independently as mutually orthogonal attributes with proper correspondences of hue with hue angle, lightness with metric lightness and chroma with metric chroma in the new colour space NC-IIIC.

KEYWORDS: non-linear uniform colour space, opponent colour response mechanisms

1. Introduction

The colour space is requested to satisfy the following conditions for specifying colour in accordance with a common standard.

1) Specification of colour can be made in correspondence with physiological mechanisms of the visual system.
2) Metric quantities can represent psychological aspects of colour corresponding to perceived attributes.
3) Geometrical distance in the space is well corresponded with physiological mechanisms of the visual system and did not include equations which express non-linearity and non-symmetry corresponding with opponent colour response mechanisms.
4) Geometrical distance in the space is well corresponded with physiological mechanisms of the visual system and did not include equations which express non-linearity and non-symmetry corresponding with opponent colour response mechanisms.
5) Mathematical and geometrical structures are clear and lucid to assure simplicity and convenience in both calculation and specification of coordinates.

On the other hand, on the colour appearance model where chromatic adaptation is considered for the purpose of specifying how colour is appeared, the above condition (1) is different and the following conditions must be fulfilled in addition to the above statements (2) to (5).

6) Hunt effect can be represented to show the fact that colour is more vividly perceived as illuminance level becomes higher.
7) Stevens effect can be expressed to show the effect that colour of higher lightness against the background is perceived brighter; colour of lower lightness is perceived darker, and this effect becomes more conspicuous as the illuminance becomes higher.
8) Helson effect can be indicated to express the phenomenon perceived as a colour of the opposite hue of light source. In other words, the colour appearance model should be capable of representing how colour is perceived under various circumstances, such as colour of illuminant, illuminance level, luminance and colour of the background, whether object is luminous or non-luminous, the size of the object, reflectance / luminance and colour of the object, area of visual field, viewing conditions and so forth. Meanwhile, when specifying colour in a standard manner, achromatic colour must be represented as achromatic under any condition, and hue, lightness and chroma of the colour must be solely defined as attributes of the object regardless of illuminance level and conditions of the background. These are the important differences between the conditions for the space to specify colour in a standard manner and those for the colour appearance model to represent how colour is appeared. The CIE has recommended the use of \( L^*a^*b^* \) uniform colour space and \( L^*u^*v^* \) uniform colour space for colour specification. Also, for colour rendering indices, the use of \( U^*V^*W^* \) space was proposed. However, these spaces did not take into consideration functions and characteristics of the opponent colour response mechanisms of the visual system and did not include equations which express non-linearity and non-symmetry corresponding with opponent responses. Therefore, it has not been possible to represent colour accurately and uniformly in correspondence with...
attributes of perceived colour, such as hue lightness, and chroma.

Up to this time, non-linearities at the receptor level are expressed by non-linear functions of $X$, $Y$, and $Z$ or $R$, $G$, and $B$ responses. Some of these non-linear functions, such as Munsell value that is used to express lightness, are complicated. Yet recently, approximation is attempted by employing functions that are as simple as possible, and functions of $1/3$ power are frequently used, for example, $L^*a^*b^*$ uniform colour space of the CIE adopts the same form of non-linear functions as well.

Non-linearity can be introduced after transforming to $R$, $G$, and $B$, however, functions of $X$, $Y$, and $Z$ to $1/3$ power are adopted in this study, because $X$, $Y$, and $Z$ are used in colour spaces of the CIE. For comparison purposes, further trial to study those to $1/2$ power is attempted.

When introducing non-linearity to linear space NC-I in conjunction with the above concept, general expression for coordinates of colour in the space is described as follows.

$$L^* = \frac{Y}{Y_n}^{1/3} - 16$$

$$a' = K\left[ \frac{X}{X_n} \right]^{1/3} - \frac{1}{3} \left( \frac{X}{X_n} \right)^{1/3}$$

$$b' = K\left[ \frac{Y}{Y_n} \right]^{1/3} - \frac{1}{3} \left( \frac{Z}{Z_n} \right)^{1/3}$$

Values of coefficient $K$ for respective colour spaces, by which mean radius of the hue circle of $C=8$ becomes 40, are shown in Table 1.

<table>
<thead>
<tr>
<th>Colour Space</th>
<th>$K$</th>
</tr>
</thead>
<tbody>
<tr>
<td>NC-II S</td>
<td>160</td>
</tr>
<tr>
<td>NC-II C</td>
<td>190</td>
</tr>
<tr>
<td>L<em>a</em>b*</td>
<td>200</td>
</tr>
</tbody>
</table>

Figure 1 Coordinates of Munsell colours with value of 6 in Judd, NC-I, Lab, Luv, NC-II S and NC-II C spaces.

The general term for these new colour spaces will be defined as NC-II (New Colour Space-II), and “S” (square root) and “C” (cubic root) will be indicated at the end for $1/2$ and $1/3$ power spaces, respectively.

Values of coefficient $K$ for respective colour spaces, by which mean radius of the hue circle of $C=8$ becomes 40, are shown in Table 1.

<table>
<thead>
<tr>
<th>Colour Space</th>
<th>$K$</th>
</tr>
</thead>
<tbody>
<tr>
<td>NC-II S</td>
<td>160</td>
</tr>
<tr>
<td>NC-II C</td>
<td>190</td>
</tr>
<tr>
<td>L<em>a</em>b*</td>
<td>200</td>
</tr>
</tbody>
</table>

 Coordinates of Munsell colours located orderly on the plane of $V=6$ in new colour spaces NC-IIS and NS-IIC are shown in Fig. 1, in comparison with those in Judd, NC-I, L*a*b* and L*a*v*colour spaces.
(2) It is proved that both non-linearity and non-symmetry in Y-B and R-G opponent colour response mechanisms can be expressed by functions of high degree singular terms.

(3) It became clear that 1/3 power functions are superior to 1/2 power functions in expressing non-linearity at the receptor level, and it is reconfirmed that the concept of non-linearity hitherto adopted in $L^a^*b^*$ uniform colour space and the like is appropriate.

(4) On NC-IIIC space which is derived from linear colour space NC-I, Y-B response axis and R-G response axis are geometrically perpendicular to and independent of each other. Whereas they are slanted in $L^a^*b^*$ N space derived from Judd linear space. This indicates that the term relating to $Z$ in the equation of $a^*$ that expresses R-G response can not be ignored.

8. Concluding remark

Up to this point, it has been commonly accepted that physiological colour perceptive mechanisms are constructed by three primary colour responses at the cone level and opponent colour responses at horizontal cell level, respectively, since Young-Helmholtz three-component theory, Hering opponent-colours theory and Hurvich & Jameson chromatic valence theory. Today, a unified staged theoretical colour perceptive model, which incorporates all of these theories, is widely accepted.

These perceptual responses must be considered non-linear and non-symmetric naturally, because these mechanisms are based on physiological responses. However, with regards to spaces for colour specification, there have been no equations introducing expressions of non-linearity and non-symmetry in opponent colour response mechanisms, while there were equations expressing non-linearity at the cone level.

In this study, new non-linear and non-symmetric uniform colour space NC-IIIC for colour specification has been developed successfully by applying non-linearity at the receptor level in linear space and by introducing response functions which express non-linearity and non-symmetry in opponent colour response mechanisms.

As a result, loci or hue circles of colours of constant value and constant chroma in Munsell system can be represented as concentric circles that are placed with even spacing according to the magnitude of chroma.

On reflection, above characteristics of Munsell system are extremely remarkable considering the fact that coordinates of $X$, $Y$ and $Z$ of Munsell colours were estimated manually in a period when computers were non-existent.

We should pay great respect to this prominent achievement of arranging such lots of coordinates by external and internal insertions of enormous data obtained through psychological experiments in a systematic way by human hand.

Honestly speaking, I reconfirmed the fact that Munsell system is constructed in an ultimately systematic manner, and rediscovered that it is appropriately structured to make up Euclidean uniform colour space.

In this non-linear uniform colour space, hue circles can be represented as uniform circles, and hue and chroma can be independently corresponded with hue angle or metric hue and distance from the origin or metric chroma, respectively. Hence it will be possible to specify colour in correspondence with characteristics of human colour vision.

Moreover, the change in hue and that in chroma can be independently corresponded with the change in hue angle and that of distance from the origin or in radius of hue circle, respectively. This means the colour difference can be specified in accordance with the attributes of human colour perception.

Likewise in the linear colour space, the expression by the equations can be applied to colour under any illuminant because von Kries type compensation is introduced in the non-linear colour space.

In the future, we plan to verify micro uniformity of the new space, particularly relating to hue angle, by applying this colour space to results of colour difference evaluation tests.

We also wish to make an attempt at elucidating the geometric structure of this colour space by applying the concept of non-linearity and non-symmetry in opponent colour response mechanisms to NCS colour system.

References